We have a rotation matrix $R_{12}$ and want the angular velocity $\omega_{12}$. Starting from

$$R_{12} = R_{12}(q, t), \quad q \in \mathbb{R}^{n_q}$$

(1)

we build the total derivative

$$\dot{R}_{12} = \sum_{i=1}^{n_q} \frac{\partial R_{12}}{\partial q_i} \dot{q}_i + \frac{\partial R_{12}}{\partial t}$$

(2)

Multiply from right with $R_{12}^T$

$$\dot{R}_{12} R_{12}^T = \sum_{i=1}^{n_q} \left( \frac{\partial R_{12}}{\partial q_i} R_{12}^T \right) \cdot \dot{q}_i + \frac{\partial R_{12}}{\partial t} R_{12}^T$$

(3)

Apply the inverse tilde operator, which transforms a skew symmetric matrix to a vector. This inverse tilde operator is distributive and linear.

$$\tilde{\left( \dot{R}_{12} R_{12}^T \right)} = \sum_{i=1}^{n_q} \left( \tilde{\frac{\partial R_{12}}{\partial q_i}} R_{12}^T \right) \cdot \dot{q}_i + \tilde{\frac{\partial R_{12}}{\partial t} R_{12}^T}$$

(4)

Rewrite it in matrix notation

$$\tilde{\left( \dot{R}_{12} R_{12}^T \right)} = \begin{bmatrix} \tilde{\frac{\partial R_{12}}{\partial q_1} R_{12}^T} & \tilde{\frac{\partial R_{12}}{\partial q_2} R_{12}^T} & \cdots \end{bmatrix} \cdot \dot{q} + \tilde{\frac{\partial R_{12}}{\partial t} R_{12}^T}$$

(5)

and apply some substitutions

$$\tilde{\left( \dot{R}_{12} R_{12}^T \right)} = \begin{bmatrix} \tilde{\frac{\partial R_{12}}{\partial q_1} R_{12}^T} & \tilde{\frac{\partial R_{12}}{\partial q_2} R_{12}^T} & \cdots \end{bmatrix} \cdot \dot{q} + \tilde{\frac{\partial R_{12}}{\partial t} R_{12}^T}$$

(6)

Hence if we define the partial derivative operator of an rotation matrix $R_{12}$ with respect to a scalar $x$ or a vector $x$ formally as

$$\text{parder}_x(R_{12}) := \left( \frac{\partial R_{12}}{\partial x} R_{12}^T \right)$$

(8)

$$\text{parder}_x(R_{12}) := \begin{bmatrix} \frac{\partial R_{12}}{\partial x_1} R_{12}^T & \frac{\partial R_{12}}{\partial x_2} R_{12}^T & \cdots \end{bmatrix}$$

(9)

than the rotation is fully equal to the translation and using the new fmatvec Function concept yields the following:

$$R_{12} = (\ast \text{rotFunc})(q, t) \in \mathbb{R}^{3 \times 3}$$

(10)

$$\dot{1}J_R = \text{rotFunc}\rightarrow\text{parDer1}(q, t) \in \mathbb{R}^{3 \times n_q}$$

(11)

$$\dot{1}j_R = \text{rotFunc}\rightarrow\text{parDer2}(q, t) \in \mathbb{R}^{3}$$

(12)

Just analog to the translations

$$\dot{1}r = (\ast \text{transFunc})(q, t) \in \mathbb{R}^{3}$$

(13)

$$\dot{1}J_T = \text{transFunc}\rightarrow\text{parDer1}(q, t) \in \mathbb{R}^{3 \times n_q}$$

(14)

$$\dot{1}j_T = \text{transFunc}\rightarrow\text{parDer2}(q, t) \in \mathbb{R}^{3}$$

(15)